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B.M.S. COLLEGE FOR WOMEN, AUTONOMOUS

BENGALURU – 560004

SEMESTER END EXAMINATION – SEPT/OCT 2023

M.Sc in Mathematics – 4th Semester

DESIGN AND ANALYSIS OF ALGORITHM

Course Code: MM410T **QP Code: 14010**
Duration: 3 Hours **Max marks: 70**
Instructions: ***1) All questions carry equal marks.***
2) Answer any FIVE full questions.

1. (a) Illustrate best case, average case, worst case analysis using an example. Prove that the running time of on algorithm is $O(g(m))$ if and only if its worst case running time is $O(g(n))$ and its best case running time ids $\Omega(g(n))$.

   (b) Find the time complexity of the following recurrence relation

   (i) $T(n) = 4T\left(\frac{n}{2}\right) + n$ (ii) $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

   (c) Define (i) Record (ii) Binary tree (iii) Heap (iv) Queue, with example for each.

   (5+5+4)

2. (a) Explain quick sort technique to sort the following array. Find the run time.

24	50	30	20	21	18	3	12	4	7	17	11

   (b) Search the key $x = 24$ in the following array. Explain the technique in detail along with time analysis.

-5	-2	0	3	8	8	9	12	26	31	24	18	29	42	23

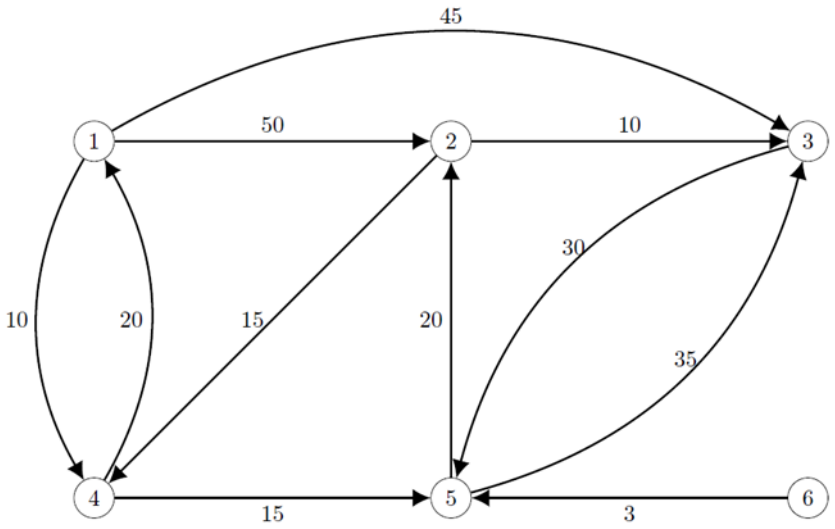
   (7+7)

3. (a) Using Radix sort algorithm, Sort the following array:

   5002, 4001, 380, 292, 65492, 41893, 90282, 2009, 29854, 1000, 504, 65495, 2017, 6665, 9078, 62000.

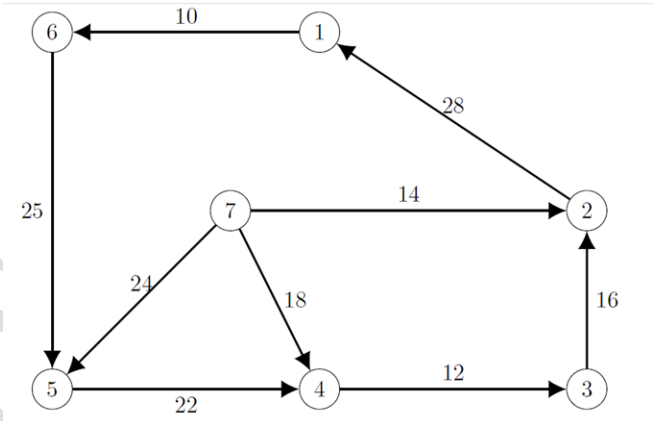
   (b) What is minimum cost spanning tree? Apply Prim's algorithm to find the minimum

spanning tree of the graph . Write the greedy approach used in this algorithm. Explain how it affects its runtime.



(7+7)

4. (a) Write Dijkstra's shortest path algorithm. Apply Dijkstra's algorithm to obtain shortest paths, treating every vertex as a source.



b) Obtain the optimal solution for the given Travelling Salesmen problem using branch and bound method.

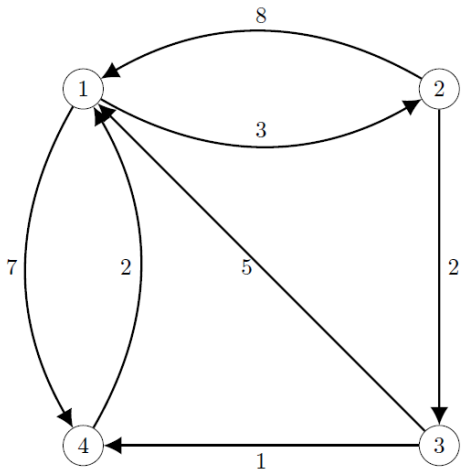
$$\begin{bmatrix} 0 & 16 & 11 & 6 \\ 8 & 0 & 13 & 16 \\ 4 & 7 & 0 & 9 \\ 5 & 12 & 2 & 0 \end{bmatrix}$$

(7+7)

5. (a) Write a short note on optimal triangulation of a polygon.

   (b) Write Floyd's algorithm and solve all the pair of shortest path problem for the graph

shown below.



(4+10)

6. Find the cost and structure of an optimal binary search free for a set of 7 keys with the following probabilities . Write the runtime.

i	0	1	2	3	4	5	6
p_i	-	0.04	0.06	0.08	0.02	0.1	0.12
q_i	0.06	0.06	0.06	0.05	0.05	0.06	0.05

(14)

7. Find the matrix chain multiplication under and apply it for the following sequence of dimensions $\langle 4, 6, 8, 7, 3, 9 \rangle$. Also give complete parenthesis and runtime. **(14)**

8. (a) What are reduction algorithms? Prove that the subset sum problem can be reduced to the job-scheduling problem.

   (b) Consider the language HAM-CYCLE = $\{(G)/\ G\ is\ a\ Hamiltonian\ graph\}$.

   Prove that HAM-CYCLE is a NP problem by describing a polynomial time algorithm to verify the language.

(7+7)